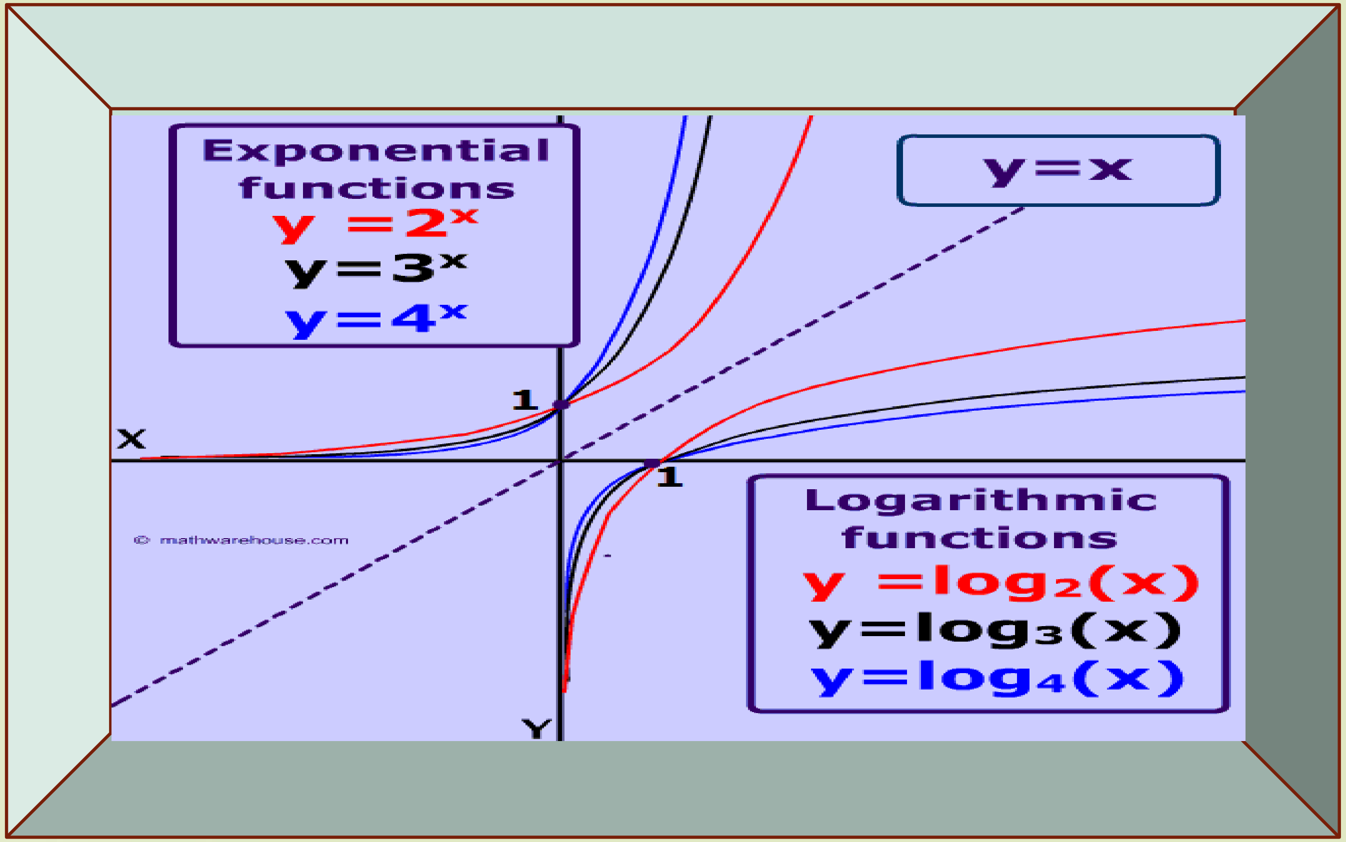
# CONTINUITY & DIFFERENTIABILITY-MODULE 4

TOPICS

- ❖ EXPONENTIAL AND LOGARITHMIC FUNCTIONS
- ❖ RULES RELATED TO THESE FUNCTIONS
- ❖ DERIVATIVES



# WHAT'S LOGHARITHMIC &EXPONENTIAL fn:

A function of the form $f(x) = \log_a x$ (where $a > 0$ and $a \neq 1$) is called a logarithm function.

A function of the form $f(x) = a^x$ (where $a > 0$) is called an exponential function.

Particularly important exponental function is $f(x) = e^X$, where
$e = 2.718 \ldots$. This is often called 'the' exponential function

A particularly important logarithm function is $f(x) = \log_e x$, where $e = 2.718\ldots$. This is often called the natural logarithm function, and written $f(x) = \ln x$.

COMMON LOG.

$10^4 = 10000$: $\log_{10} 1000 = 4$; ; $10^{-4} = 0.0001$: $\log_{10} 0.0001 = -4$

# Properties Of Logarithm

**Logarithm Properties**

$$\log_a xy = \log_a x + \log_a y$$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$\log_a x^n = n \log_a x$$

$$\log_a b = \frac{\log_c b}{\log_c a}$$

$$\log_a b = \frac{1}{\log_b a}$$

Log 1 = 0

The following can be derived from the above properties.

$$\log_a 1 = 0$$

$$\log_a a = 1$$

$$\log_a a^r = r$$

$$\log_a \frac{1}{b} = -\log_a b$$

$$\log_{\frac{1}{a}} b = -\log_a b$$

$$\log_a b \log_b c = \log_a c$$

$$\log_{a^m} a^n = \frac{n}{m}, m \neq 0$$

log 10=1

log e = 1

RULE

$$\log_a x^n = n\log_a x$$

$$\log_a xy = \log_a x + \log_a y$$

Example:

$$3^x - 1 = 4$$
$$3^x = 5$$
$$\log 3^x = \log 5$$
$$x\log 3 = \log 5$$
$$x = \frac{\log 5}{\log 3}$$

Example:

$$5^{x-1} - 2^x = 0$$
$$5^{x-1} = 2^x$$
$$\log 5^{x-1} = \log 2^x$$
$$(x-1)\log 5 = x\log 2$$
$$x\log 5 - \log 5 = x\log 2$$
$$x\log 5 - x\log 2 = \log 5$$
$$x(\log 5 - \log 2) = \log 5$$
$$x = \frac{\log 5}{\log 5 - \log 2}$$

Let $y = \cos x.\cos 2x.\cos 3x$

Taking logarithm on both the sides, we obtain

$$\log y = \log(\cos x.\cos 2x.\cos 3x)$$
$$\Rightarrow \log y = \log(\cos x) + \log(\cos 2x) + \log(\cos 3x)$$

RULE

RULE: $\log\left(\sqrt{\frac{AB}{C}}\right) = \frac{1}{2}(\log A + \log B - \log C)$

$$\log_a \frac{x}{y} = \log_a x - \log_a y$$

$$y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

taking log on both sides

$$\log y = \log\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

$$\log y = \frac{1}{2}[\log(x-1) + \log(x-2) - \log(x-3) - \log(x-4) - \log(x-5)]$$

$$Y = \frac{x-1}{x+5}$$

Taking log. Both sides

logy = log(x-1)-log(x+5)

$Y = e^{\sin^{-1}x}$

Applying log. On both sides

$\log y = \sin^{-1}x.\log e$

$\log y = \sin^{-1}x.1$

Differentiate w.r.t x

$$\frac{1}{y}.\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = y.\frac{1}{\sqrt{1-x^2}}$$

$$\frac{dy}{dx} = e^{\sin^{-1}x}.\frac{1}{\sqrt{1-x^2}}$$

# HOW DO LOG. HELPS YOU IN DERIVATIVES?

$y = x^n$

- Taking log.on both sides
- log y = n log x
- Differentiating both sides
- $\frac{1}{y}\frac{dy}{dx} = n.\frac{1}{x}$
- $\frac{dy}{dx} = y\,(n.\frac{1}{x})$
- $\frac{dy}{dx} = x^n(n.\frac{1}{x}) = n\,x^{n-1}$

# Applying Log.Rule & differentiate

- $y = \log\left(x + \sqrt{x^2 + 1}\right)^2$
- APPLY LOG. RULE
- $y = 2.\log\left(x + \sqrt{x^2 + 1}\right)$
- Differentiate w.r.t x
- $\frac{dy}{dx} = 2.\frac{1}{x+\sqrt{x^2+1}}\left(1 + \frac{1}{2\sqrt{x^2+1}} \times 2x\right)$
- $= 2.\frac{1}{x+\sqrt{x^2+1}}\left(\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}\right)$
- $= 2\frac{1}{\sqrt{x^2+1}} = \frac{2}{\sqrt{x^2+1}}$

WITH CHAIN RULE

# APPLY LOG. ON BOTH SIDES

- $(\sin x)^y = (\sin y)^x$
- $Y = 2^{\sin x}$
- $Y = x^{\log x}$
- $Y = \sqrt{(x-5)(x+8)}$
- $xy = e^{x-y}$
- $y = (x-2)^3(2x-9)^4$

Use log. rules and answer quickly

. . y = (3x + 5) $^{(2x - 3)}$

**Solution:**

$We\ know\ that$

$$y = (3x+5)^{2x-3}$$

$By\ taking\ log\ on\ both\ sides$

$$log y = log((3x+5)^{2x-3})$$

$It\ can\ be\ written\ as$

$$log y = (2x-3) log(3x+5)$$

$By\ differentiating\ both\ sides\ w.r.t.x$

$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = (2x-3).\frac{\mathrm{d}[log(3x+5)]}{\mathrm{d}(3x+5)} \times \frac{\mathrm{d}(3x+5)}{\mathrm{d}x} + log(3x+5)\frac{\mathrm{d}(2x-3)}{\mathrm{d}x}$$

$On\ further\ calculation$

$$\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x} = (2x-3).\frac{1}{3x+5}.3 + log(3x+5).2$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = y(\frac{3(2x-3)}{3x+5} + 2log(3x+5))$$

$Substituting\ the\ value\ of\ y$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = (3x+5)^{2x-3}[\frac{3(2x-3)}{3x+5} + 2log(3x+5)]$$